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# Validity of Describing Resonant Viscosity and Mass Density Sensors by Linear 2nd Order Resonators

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## Abstract

Resonant viscosity and mass density sensors are based on the determination of a mechanical oscillator's frequency response at a characteristic resonant mode upon immersion in a liquid. In many cases the sensors' characteristics and sensitivities are demonstrated by showing the dependence of their quality factor  $Q$  and resonance frequency  $f_r$  to viscosity  $\eta$  and mass density  $\rho$ . Commonly, the transfer function of a linear second order resonator is fit into the recorded frequency response to extract  $f_r$  and  $Q$ , which are then related to the liquid's  $\eta$  and  $\rho$ . This approach is widespread in the field of resonant (viscosity and mass density) sensors and thus became a common procedure. However, as from a theoretic point of view, resonators which are interacting with a liquid do not yield linear second order functions, the applicability of this standard approach and its limits are investigated in this work.

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**Keywords:** viscosity, mass density, resonator, quality factor, resonance frequency;

## 1. Introduction

Resonant viscosity and mass density sensors are very attractive devices for fluid characterization [1]. The high interest for such devices is not only remarkable due to the high number of reported principles during the past two decades rather than their ongoing and still increasing amount of publications. In most cases, the change of resonance frequency  $f_r$  and quality factor  $Q$  upon immersion in a liquid is determined and related to the liquid's viscosity  $\eta$  and mass density  $\rho$ . Examples for such liquid loaded resonators (LLRs) for which data for  $Q$  and  $f_r$ ,  $\eta$  and  $\rho$  are provided are, e.g., a Si-cantilever [2], a Si-platelet [3], a quartz tuning fork [4], a U-shaped wire [5] as well as steel tuning forks with circular and rectangular cross-sections and a spiral spring resonator, which can be found in these proceedings. We want to briefly describe this common approach and highlight possible breaches and associated limits of this standard procedure. We consider the resonant device (i.e. the viscosity and mass density sensor) being representable by a linear, mechanical oscillator, see Fig.1, with lumped

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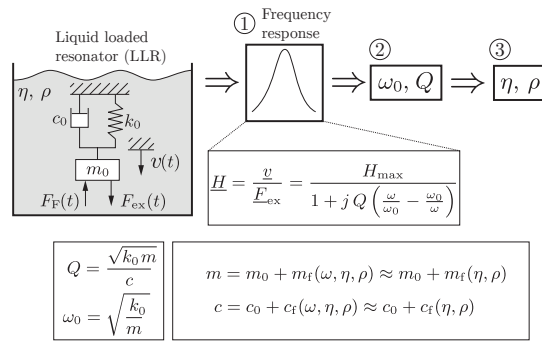


Figure 1: Principle idea of a liquid loaded resonator for viscosity and mass density measurements

mass  $m_0$ , damping coefficient  $c_0$  and spring constant  $k_0$ , interacting with a liquid. The frequency response containing a characteristic resonance is recorded. Assuming the applicability of the transfer function of a second order resonator,  $f_r$  and  $Q$  are evaluated with an appropriate second order fitting algorithm e.g., [6]. With the knowledge of these two parameters,  $\eta$  and  $\rho$  can be determined with an associated model. For this approach it is assumed, that the force  $F_F$  induced by the fluid can be represented by a complex valued function in the frequency domain (with angular frequency  $\omega$ ). As the interaction will have a linear character for small vibration amplitudes  $F_F$  will be proportional to the velocity  $v$  such that the fluid force can be related to an acoustic impedance  $\underline{Z}(\omega)$  and thus,  $\underline{F}_F = \underline{Z}(\omega) \cdot \underline{v}$ . (Note underlined variables denote complex quantities.) The acoustic impedance can be split in a real and an imaginary part which suggests to write  $\underline{F}_F = (c_f + j\omega m_f) \cdot \underline{v}$ , where  $c_f$  and  $m_f$  are the fluid loading related added damping and mass parameter, respectively. In general  $c_f$  and  $m_f$  will be frequency dependent and thus, the description as a 2nd order system with constant parameters is an approximation. However, in the small captured bandwidth this frequency dependence might be negligible.

## 2. Liquid loaded resonator (LLR)

We assume that the resonance spectrum is ideally recorded, i.e. any background signals (including noise) can be perfectly subtracted, each measured point gives a steady state value and the measured signal is not subjected to non-linearity effects. A common approach for modeling a liquid loaded resonator is to model the effect of the liquid on the resonance characteristics, (i.e.  $f_r$  and  $Q$ ) by taking an additional fluid related mass  $m_f$  and damping  $c_f$  into account. In [7] it was found that

$$\begin{aligned} m_f(\omega, \eta, \rho) &= C_{m1} \rho + C_{m2} \sqrt{\frac{\eta \rho}{\omega}} \quad \text{and} \\ c_f(\omega, \eta, \rho) &= C_{c1} \eta + C_{c2} \sqrt{\omega \eta \rho} \end{aligned} \quad (1)$$

where the factors  $C_{ji}$  are given in Tab. 1 for the case of an in-plane oscillating plate, an oscillating sphere and a laterally oscillating cylinder. Substitution of  $C_{m2} = \sqrt{\omega_0} C_{m2}^*$  and  $C_{c2} = \frac{C_{c2}^*}{\sqrt{\omega_0}}$  (where  $\omega_0$  is the angular resonance frequency) in Eq. 1

|          | Plate                  | Sphere                 | Cylinder                 |
|----------|------------------------|------------------------|--------------------------|
| $C_{m1}$ | 0                      | $\frac{2\pi}{3} r_s^3$ | $\pi r_c^2 l_c$          |
| $C_{m2}$ | $\frac{A_p}{\sqrt{2}}$ | $\sqrt{2} 3 \pi r_s^2$ | $\sqrt{2} 2 \pi r_c l_c$ |
| $C_{c1}$ | 0                      | $6 \pi r_s$            | $2 \pi l_c$              |
| $C_{c2}$ | $\frac{A_p}{\sqrt{2}}$ | $\sqrt{2} 3 \pi r_s^2$ | $\sqrt{2} 2 \pi r_c l_c$ |

Table 1: Coefficients in the apparent fluid mass and damping parameters Eq. 1 for the case of an in-plane oscillating plate, an out-of-plane oscillating sphere and cylinder.  $r_s$ : sphere radius,  $r_c$ : cylinder radius,  $l_c$ : cylinder length,  $A_s$ : surface of the paltelet

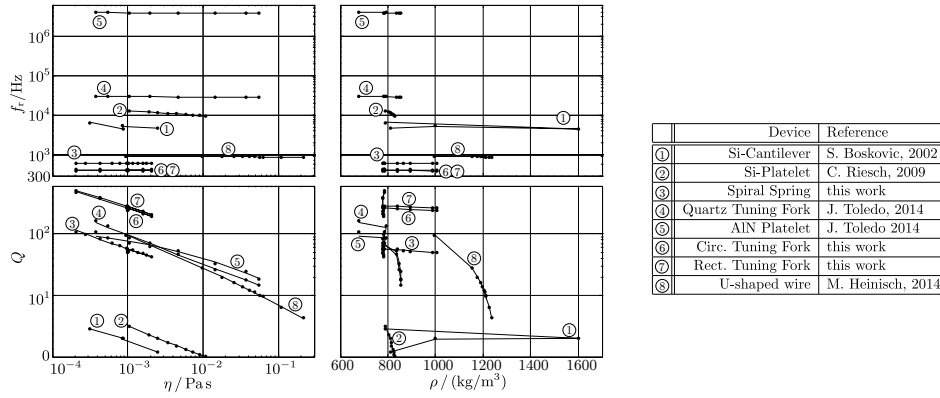


Figure 2: Selection of liquid loaded resonators

and assuming that the investigated frequency range around the resonance is small (i.e.,  $\omega \approx \omega_0$ ) it follows that

$$\begin{aligned} m_f(\eta, \rho) &\approx C_{m_1} \rho + C_{m_2}^* \sqrt{\eta \rho} \quad \text{and} \\ c_f(\eta, \rho) &\approx C_{c_1} \eta + C_{c_2}^* \sqrt{\eta \rho}. \end{aligned} \quad (2)$$

With these expressions for  $m_f$  and  $c_f$  it follows for  $\omega_0$  and  $Q$ :

$$\begin{aligned} \frac{1}{\omega_0^2} = \frac{m}{k_0} &\approx m_{0k} + m_{\rho k} \rho + m_{\eta \rho k}^* \sqrt{\eta \rho}, \quad \text{and} \\ \frac{1}{Q} = \frac{c}{\sqrt{k_0 m}} &\approx \frac{c_{0k} + c_{\eta k} \eta + c_{\eta \rho k}^* \sqrt{\eta \rho}}{\sqrt{m_{0k} + m_{\rho k} \rho + m_{\eta \rho k}^* \sqrt{\eta \rho}}} \approx D_0 + D_\eta \eta + D_{\eta \rho} \sqrt{\eta \rho}. \end{aligned} \quad (3)$$

Although assumed to be almost constant at this point, in Sec. 4 the change of  $m_f$  and  $c_f$  in the investigated frequency range will be studied. The major advantages of such a model are its general applicability to many different resonator principles, (i.e. also principles other than in-plane oscillating plates, oscillating spheres and laterally oscillating cylinders), the intuitive descriptiveness of liquid loaded resonators and thus the possibility of comparing different types of viscosity and mass density sensors. Showing the characteristics of such sensors by plotting e.g.  $Q$  versus  $\eta$  might be misleading or giving the values in tabulated form does not allow an insightful description of the device, as  $\eta$  and  $\rho$  both have significant effects on  $f_r$  and  $Q$ . Furthermore, giving a sensitivity of the devices is difficult, as the sensitivities to  $\eta$  and  $\rho$  are not constant but strongly  $\eta$  and  $\rho$  dependent. With the knowledge of the parameters of this model, the sensors characteristics are usually very well described.

### 3. Examples of liquid loaded resonators

The model for  $f_r = \frac{\omega_0}{2\pi}$  and  $Q$ , Eq. 3, was applied to liquid loaded resonators which were found in a comprehensive literature review if sufficient data (i.e.  $f_r$  and  $Q$  with associated  $\eta$  and  $\rho$  in tabulated form) was provided. The resonators were selected such that operational frequencies in a range from 100s of Hertz to the Mega Hertz range and quality factors roughly between 1 and 500 in liquids can be investigated. With the data found in literature and from our own work, the parameters in the model (Eq. 3) were determined performing a first linear LSQ fit for determining the parameters  $m_{0k}, m_{\rho k}, m_{\eta \rho k}^*$  and a subsequent LSQ fit for determining the parameters  $c_{0k}, c_{\eta k}, c_{\eta \rho k}^*$ . An overview of the characterized LLRs is shown in Fig. 2 on a double logarithmic scale for  $f_r$  and  $Q$  and associated  $\eta$  and  $\rho$ . There, the dots indicate the reported values and the solid lines are the results from the fitted model.

### 4. Estimation of the change of $m(\omega)$ and $c(\omega)$ in the investigated frequency range

To estimate the change of  $m(\omega) = m_0 + m_f(\omega, \eta, \rho)$  and  $c(\omega) = c_0 + c_f(\omega, \eta, \rho)$  in the investigated bandwidth  $[\omega_0 - N_{\delta l} \cdot \delta, \omega_0 + N_{\delta r} \cdot \delta]$  (where  $\delta = \omega_0/(2Q)$  and  $N_{\delta l, r}$  are preset numbers for defining starting and ending frequencies) a

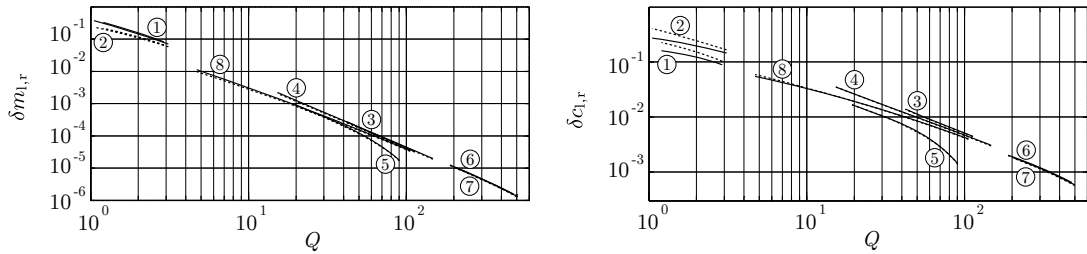


Figure 3: Relative changes of  $m(\omega)$  and  $c(\omega)$  in the investigated frequency range for different quality factors.

frequency dependence is re-introduced into the model by substituting  $m_{\eta\rho k}^*$  by  $m_{\eta\rho k}^* \sqrt{\omega_0}/\sqrt{\omega}$  and  $c_{\eta\rho k}^*$  by  $c_{\eta\rho k}^* \sqrt{\omega}/\sqrt{\omega_0}$ , respectively yielding

$$\begin{aligned} \frac{m(\omega)}{k_0} &\approx m_{0k} + m_{\rho k} \rho + \sqrt{\omega_0} m_{\eta\rho k}^* \sqrt{\frac{\eta\rho}{\omega}}, \quad \text{and} \\ \frac{c(\omega)}{k_0} &\approx c_{0k} + c_{\eta k} \eta + \frac{c_{\eta\rho k}^*}{\sqrt{\omega_0}} \sqrt{\omega \eta \rho}. \end{aligned} \quad (4)$$

With this, the relative changes of mass and damping parameter  $\delta m_{l,r}$  and  $\delta c_{l,r}$ , respectively are calculated which become dependent on  $Q$  only, if  $\delta = \omega_0/(2Q)$  is substituted. In Fig. 3 these relative changes are depicted for  $N_{\delta l} = N_{\delta r} = 2$  for the examples of liquid loaded resonators shown in Fig. 2. There, it can be observed, that for quality factors  $Q > 100$  the change of the liquid loaded parameters  $m(\omega)$  and  $c(\omega)$  is negligibly small, but for  $Q < 10$  it can become larger than 10 %.

## 5. Conclusion and Outlook

A generalized model for  $f_r$  and  $Q$  was applied to the data found in literature. With the knowledge of these fitted parameters a frequency dependence was re-introduced in this initially frequency-independent model. With this frequency dependent model the changes of the liquid loaded parameters  $m(\omega)$  and  $c(\omega)$  could be estimated. By this approach it could be estimated, that these changes can become larger than 10 % for  $Q < 10$ . Regarding future work, this first investigation of the validity of describing LLRs with second order models will be extended, including the theory of deriving second order models from eigenmode functions of vibrating structures, a guideline for a measuring procedure ensuring linear and steady state deflections and an estimation of the error on  $\eta$  and  $\rho$  which results from the systematic error of fitting a second order transfer function into the frequency response of a LLR.

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